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VIBRATION ANALYSIS OF THREE PARAMETER MODEL OF ADHESIVELY BONDED JOINTS

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ABSTRACT

An analytical model to study the coupled transverse and longitudinal vibrations of a single lap adhesive joint is proposed in this paper which includes partial differential form of the motion equations. A balanced single lap adhesive joint consist of two identical adherents of mild steel which are lap jointed over a certain length by a viscoelastic material, epoxy resin(araldite). Adherents are modeled as Euler-Bernoulli Free-Free beam. Both transverse and axial deformation of adherents, shear and peel stresses at the adhesive joint interface and deflection of mid plane of adhesive layer (3-parameter model) is considered in deriving the equations of motion. The classical two parameter elastic foundation model violates the equilibrium condition of the adhesive layer; to eliminate this flaw, a new three parameter elastic foundation model is considered which satisfies the equilibrium condition of the adhesive layer. The governing equations of motions are derived for three parameter elastic foundation model. The numerical solutions of the governing equations for free vibrations yield the system natural frequency. Experimentation carried out on both monolithic and adhesively jointed beam to observe the effect of joint. The effect of thickness of adhesive and joint overlap ratio on system natural frequencies are investigated.

KEYWORDS: Adhesive Joint, Euler-Bernoulli Beam, Natural Frequency, Monolithic Beam

INTRODUCTION

Joining components by using adhesives is becoming more popular with the development of adhesives with high adhesion properties. In many structures such as those for flight, automobiles and space vehicles etc. adhesively bonded structures have often been used recently because of the great advances in adhesive bonding techniques. Single lap joints are used extensively for adhesively bonded test specimens and in various configurations for wide range of industrial products.

The paper by Goland and Reissner [1] is regarded as a classical work in the area of static analysis of a single lap adhesive joint. They derived equations for evaluating the shearing and normal stresses in the bond layer; peel and shear stresses were assumed to be constants across the adhesive thickness. A comprehensive literature survey on this subject has been presented in [2, 3, 4, 5, 12, 13, 15]. Saito and Tani [6] have derived equation for predicting the modal parameters of the coupled longitudinal and flexural vibrations of a system consisting of a pair of elastic beams fixed at both ends; the longitudinal deformation of the adhesive layer is neglected in deriving the equations of motion. He and Rao [8, 9] developed an analytical model to study vibrations of bonded joints subjected to transverse loadings. The unjointed ends of the beams were assumed to be simply supported. The equation of motion was developed using Hamilton’s principle. Vaziri A, N. Hashemi [10] developed a model to understand the effects of defects such as a void in the overlap on the system response. As described in [6, 8, 9, 10], the peel stresses acting on top and bottom surfaces of adhesive layer are assumed identical, which violates the equilibrium condition of the adhesive layer. The paper [16] deals with free vibration analyses of adhesively bonded plates. The eight-node isoparametric plate element was used to model the adherends, they show the influence of patch size on the natural frequencies and mode shapes of the five-layer cantilever bonded plate with a stepped type. A three-dimensional free vibration and stress analysis of an adhesively bonded functionally graded single lap joint was carried out by [17]. They show that the effect of the adhesive material properties, were negligible on the first ten natural frequencies and mode shapes of the adhesive joint. Both the finite element method and the back propagation artificial neural network (ANN) method were used to investigate the effects of the geometrical parameters. The influence of adhesive properties on the transverse free vibration characteristics of single lap jointed cantilevered beams were studied using FEM [18], they show that the transverse natural frequencies of beam increase with increase in Young's modulus of adhesive where as poisson's ratio has little effect on frequencies. T-C Ko, C-C Lin, R-C Chu [19] used an isoparametric adhesive interface element in the vibration analysis of adhesively bonded structures. In their investigation, the natural frequencies of laminated plates with a lap joint were found to be reasonable when compared with the results for plates without joints. Gang Li, P. Lee-Sullivan [20] used a geometrically nonlinear two-dimensional FE method to study the adhesively bonded balanced single-lap joints. The effects of plane strain and plane stress conditions, boundary conditions at the adherent ends, filleted and unfilleted overlap end geometries and two different adhesive materials on the bending moment factor and the adhesive stresses have been evaluated. A three-parameter, elastic foundation model is proposed by [21, 22] to analyze interface stresses of adhesively bonded joints

In this study a novel three parameter elastic foundation model of adhesive joint is considered. The objective of the paper is to examine the effects of single lap adhesive joint on the modal parameters of the structure. A deflection of mid plane of adhesive layer (3-parameter model) is considered in deriving the equations of motion; the classical two parameter elastic foundation model violates the equilibrium condition of the adhesive layer; to eliminate this flaw, a new three parameter elastic foundation model is considered which satisfies the equilibrium condition of the adhesive layer. A beam with Free-Free configuration is considered in analytical and experimental investigation.

THEORETICAL INVESTIGATION

Single Lap Adhesive Joint

For the vibration analysis of adhesively bonded lap joint, the following assumptions are made: (1) The beams are purely elastic and the adhesive is linearly viscoelastic. (2) The system deforms in plain stress or plain strain and the usual Euler-Bernoulli beam theory is considered. (3) Shear stress in the bond material varies linearly through the adhesive thickness. (4) The adhesive layer usually has much lower extensional and bending stiffness than those of the adherents; therefore the axial and bending moment of the adhesive layer can be ignored.

Schematic diagram of a balanced single lap adhesive joint is shown in Figure 1. The mild steel adherents are modeled as Euler Bernoulli beams supported on viscoelastic foundation, which resist both peeling and shear deformations. The beams are purely elastic and have thickness t_1, width w, length (l+c), density ρ and Young's modulus E. The adhesive is considered to be viscoelastic and its complex shear and elastic modulus is assumed to be G_a and E_a respectively, h_0 is the adhesive thickness.

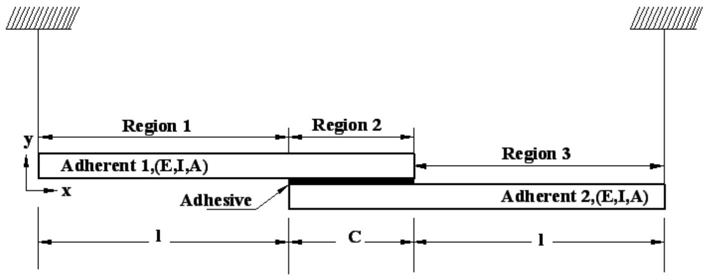

Figure 1: Schematic Model for a Single Lap Adhesive Joint

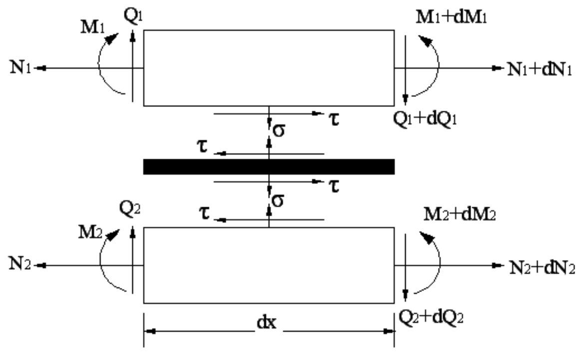

Figure 2: Free Body Diagram of an Element in Region 2

Considering the free body diagram of an element in region 2 (jointed region), (Figure 2), the equations of motion for each beam element are:

For Adherent 1

$$\frac{\partial Q_1}{\partial x} + \sigma_1 + (\rho A)\frac{\partial^2 y_1}{\partial t^2} = 0 \tag{1}$$

$$Q_1 - \frac{\partial M_1}{\partial x} + \frac{t_1}{2}\tau w = 0 \tag{2}$$

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u_1}{\partial x}\right) - \tau w - (\rho A)\frac{\partial^2 u_1}{\partial t^2} = 0 \tag{3}$$

For Adherent 2

$$\frac{\partial Q_2}{\partial x} - \sigma_2 + (\rho A)\frac{\partial^2 y_2}{\partial t^2} = 0 \tag{4}$$

$$Q_2 - \frac{\partial M_2}{\partial x} + \frac{t_1}{2}\tau w = 0 \tag{5}$$

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u_2}{\partial x}\right) + \tau w - (\rho A)\frac{\partial^2 u_2}{\partial t^2} = 0 \tag{6}$$

Where, Q_q is the shear force and M_q (q =1,2) the bending moment at the edge of the joint element, A is the cross sectional area of the beam, u_1, u_2, y_1 *and* y_2 are the longitudinal and transverse displacements of adherents. Figure3 shows a three-parameter elastic foundation model for adhesive layer.

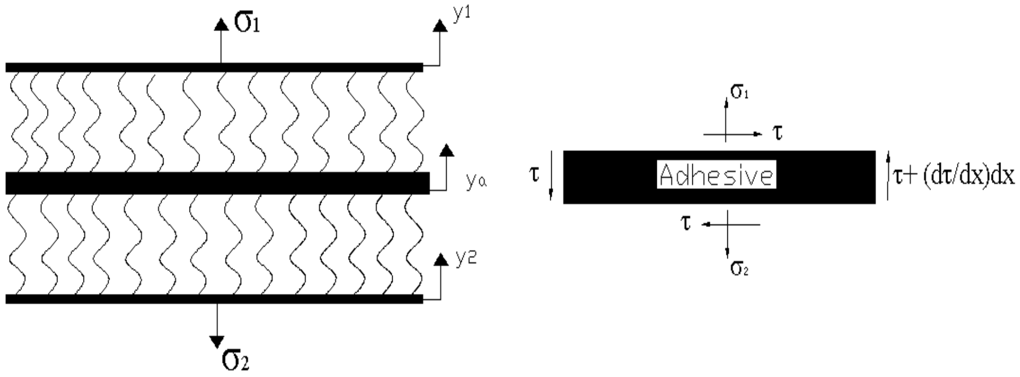

Figure 3: Three-Parameter Elastic Foundation Model for Adhesive Layer

In a two parameter model of adhesive layer ; the peel stresses acting on the top and bottom surface of the adhesive layer are assumed identical i.e. $\sigma_1 = \sigma_2$ then the equilibrium condition of the adhesive layer gives $\frac{d\tau}{dx} = 0$; this equation suggests that the shear stress is constant along the x-direction which contradicts the shear stress distribution predicted by two-parameter elastic foundation model [21]. To remove this contradiction we can assume that $\sigma_1 \neq \sigma_2$

Then we have,

$$\frac{d\tau}{dx} = \frac{\sigma_2 - \sigma_1}{h_0} \neq 0 \tag{7}$$

Shear stress in the bonded area considering longitudinal and transverse displacement of adherent and deflection y_a of mid plane of adhesive layer can be expresses as [21].

$$\tau = \frac{G_a}{2h_0}\left[2u_1 - 2u_2 + t_1\left(\frac{\partial y_1}{\partial x} + \frac{\partial y_2}{\partial x}\right)\right] + G_a \frac{\partial y_a}{\partial x} \tag{8}$$

And the adhesive peel stresses can be expresses as[21] -

$$\sigma_1 = \frac{2E_a w}{h_0}(y_1 - y_a)\ ;\ \sigma_2 = \frac{2E_a w}{h_0}(y_a - y_2) \tag{9}$$

The complex shear modulus $G_a = G'(1+\eta_{ag})$ and complex elastic modulus $E_a = E'(1+\eta_{ae})$, where G' and E' are the storage moduli and η_{ag} and η_{ae} are the adhesive loss factors.

Longitudinal forces and moments in the beams from beam theory are -

$$N_q = EA(\partial u_q / \partial x) \quad \text{and} \quad M_q = EI(\partial^2 y_q / \partial x^2) \quad , q = 1,2 \tag{10}$$

Now, from equations 3 and 8 we have

$$EA\left(\frac{\partial^2 u_1}{\partial x^2}\right) - (\rho A)\left(\frac{\partial^2 u_1}{\partial t^2}\right) - \frac{G_a w}{2h_0}\left[2u_1 - 2u_2 + t_1\left(\frac{\partial y_1}{\partial x} + \frac{\partial y_2}{\partial x}\right)\right] - G_a w\frac{\partial y_a}{\partial x} = 0 \tag{11}$$

Combining equations 1, 2, 9 and10 and simplifying gives,

$$\left\{EI\left(\frac{\partial^4 y_1}{\partial x^4}\right)+(\rho A)\left(\frac{\partial^2 y_1}{\partial t^2}\right)-\frac{G_a w t_1}{4h_0}\left[2\left(\frac{\partial u_1}{\partial x}-\frac{\partial u_2}{\partial x}\right)+t_1\left(\frac{\partial^2 y_1}{\partial x^2}+\frac{\partial^2 y_2}{\partial x^2}\right)\right]-\frac{G_a w t_1}{2}\frac{\partial^2 y_a}{\partial x^2}+\frac{2E_a w}{h_0}(y_1-y_a)\right\}=0 \tag{12}$$

Similarly, from equations 6 and 8 we have

$$EA\left(\frac{\partial^2 u_2}{\partial x^2}\right)-(\rho A)\left(\frac{\partial^2 u_2}{\partial t^2}\right)+\frac{G_a w}{2h_0}\left[2u_1-2u_2+t_1\left(\frac{\partial y_1}{\partial x}+\frac{\partial y_2}{\partial x}\right)\right]+G_a w\frac{\partial y_a}{\partial x}=0 \tag{13}$$

Combining equations 4,5, 9 and 10 and simplifying gives,

$$\left\{EI\left(\frac{\partial^4 y_2}{\partial x^4}\right)+(\rho A)\left(\frac{\partial^2 y_2}{\partial t^2}\right)-\frac{G_a w t_1}{4h_0}\left[2\left(\frac{\partial u_1}{\partial x}-\frac{\partial u_2}{\partial x}\right)+t_1\left(\frac{\partial^2 y_1}{\partial x^2}+\frac{\partial^2 y_2}{\partial x^2}\right)\right]-\frac{G_a w t_1}{2}\frac{\partial^2 y_a}{\partial x^2}-\frac{2E_a w}{h_0}(y_a-y_2)\right\}=0 \tag{14}$$

Considering the equilibrium of adhesive layer, equation 7, 8 and 9 gives

$$\frac{G_a}{2}\left[2\left(\frac{\partial u_1}{\partial x}-\frac{\partial u_2}{\partial x}\right)+t_1\left(\frac{\partial^2 y_1}{\partial x^2}+\frac{\partial^2 y_2}{\partial x^2}\right)\right]+G_a h_0\frac{\partial^2 y_a}{\partial x^2}-\frac{2E_a}{h_0}[2y_a-(y_1+y_2)]=0 \tag{15}$$

Now assuming that the beam is excited with initial harmonic excitation of frequency ω, the displacement field is – $u_j(x,t)=U_j(x).\ e^{i\omega t}$ and $y_r(x,t)=Y_r(x).\ e^{i\omega t}$; j=1,2 and r=1,2,a. The equations 11 to 15 becomes –

$$EA\left(\frac{d^2U_1}{dx^2}\right)-(\rho A\omega^2 U_1)-\frac{G_a w}{2h_0}\left[2U_1-2U_2+t_1\left(\frac{dY_1}{dx}+\frac{dY_2}{dx}\right)\right]-G_a w\frac{dY_a}{dx}=0 \tag{16}$$

$$\left\{EI\left(\frac{d^4Y_1}{dx^4}\right)-(\rho A\omega^2 Y_1)-\frac{G_a w t_1}{4h_0}\left[2\left(\frac{dU_1}{dx}-\frac{dU_2}{dx}\right)+t_1\left(\frac{d^2Y_1}{dx^2}+\frac{d^2Y_2}{dx^2}\right)\right]-\frac{G_a w t_1}{2}\frac{d^2Y_a}{dx^2}+\frac{2E_a w}{h_0}(Y_1-Y_a)\right\}=0 \tag{17}$$

$$EA\left(\frac{d^2U_2}{dx^2}\right)+(\rho A\omega^2 U_2)+\frac{G_a w}{2h_0}\left[2U_1-2U_2+t_1\left(\frac{dY_1}{dx}+\frac{dY_2}{dx}\right)\right]+G_a w\frac{dY_a}{dx}=0 \tag{18}$$

$$\left\{EI\left(\frac{d^4Y_2}{dx^4}\right)-(\rho A\omega^2 Y_2)-\frac{G_a w t_1}{4h_0}\left[2\left(\frac{dU_1}{dx}-\frac{dU_2}{dx}\right)+t_1\left(\frac{d^2Y_1}{dx^2}+\frac{d^2Y_2}{dx^2}\right)\right]-\frac{G_a w t_1}{2}\frac{d^2Y_a}{dx^2}-\frac{2E_a w}{h_0}(Y_a-Y_2)\right\}=0 \tag{19}$$

$$\frac{G_a}{2}\left[2\left(\frac{dU_1}{dx}-\frac{dU_2}{dx}\right)+t_1\left(\frac{d^2Y_1}{dx^2}+\frac{d^2Y_2}{dx^2}\right)\right]+G_a h_0\frac{d^2Y_a}{dx^2}-\frac{2E_a}{h_0}[2Y_a-(Y_1+Y_2)]=0 \tag{20}$$

In the usual fashion above equations (16 to 20) are transformed into five algebraic equations by assuming

$$U_j=Z_n.e^{\lambda_n x} \quad \text{and} \quad Y_q=Z^*{}_n.e^{\lambda_n x} \quad \text{where ; j =1,2 and q=1,2,a} \tag{21}$$

Where Z_n and Z_n^* are the arbitrary constants to be determined from boundary conditions. The equations (16 to 20) and (21) can be written in matrix form as-

$[J]_{5x5}\{V\}=0$ (22)

where $\{V\}=[Z_1\ Z_2\ Z_1^*\ Z_2^*\ Z_a^*]^T$ and $[J]_{5x5}=[a_{ij}]_{5x5}$ where,

$$a_{11}=a_{22}=\left(EA\lambda^2+\rho A\omega^2-\frac{G_a w}{h_0}\right);\ a_{12}=a_{21}=\left(\frac{G_a w}{h_0}\right);$$

$$a_{13}=a_{14}=-a_{23}=-a_{24}=a_{31}=-a_{32}=a_{41}=-a_{42}=\left(-\frac{G_a w t_1\lambda}{2h_0}\right)$$

$$a_{15}=-a_{25}=(-G_a w\lambda);a_{51}=-a_{52}=(G_a\lambda);\qquad a_{33}=a_{44}=\left(EI\lambda^4-\rho A\omega^2-\frac{G_a w{t_1}^2\lambda^2}{4h_0}+\frac{2E_a w}{h_0}\right);$$

$$a_{34}=a_{43}=\left(-\frac{G_a w{t_1}^2\lambda^2}{4h_0}\right);\quad a_{35}=a_{45}=-\left(\frac{G_a w t_1\ \lambda^2}{2}+\frac{2E_a w}{h_0}\right)\quad;\quad a_{53}=a_{54}=\left(\frac{G_a t_1\lambda^2}{2}+\frac{2E_a}{h_0}\right);$$

$$a_{55}=\left(G_a h_0\lambda^2-\frac{4E_a}{h_0}\right);$$

For non-trivial solutions of equation (22) ,

det $[J]_{5x5}=0$ (23)

Expansion of equation (23) gives fourteenth degree polynomial in λ, coefficient of which includes known geometrical and material properties of adherents and adhesive with only unknown parameter ω. The 14 roots of the polynomial are λ_n,(n=1,2….14).

Now, $U_j=\sum_1^{14} Z_n.e^{\lambda_n x}$; $Y_q=\sum_1^{14} Z^*{}_n.e^{\lambda_n x}$ j =1,2 and q=1,2,a (24)

For unbonded part of the upper and lower beam i.e. region 1 and region 3, the equilibrium equations are those of classical Euler-Bernoulli beam equations (Figure 4). The equations of motion of simple beams (unbonded part) for transverse and longitudinal vibrations are –

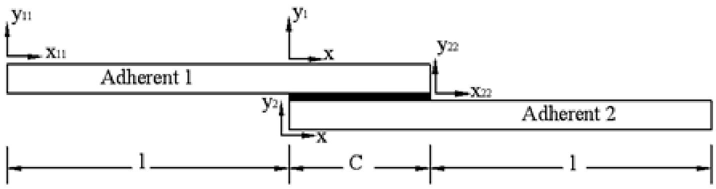

Figure 4: Bonded Joint System and Co-Ordinates

$$EI\left(\frac{\partial^4 y_{11}}{\partial {x_{11}}^4}\right)+(\rho A)\left(\frac{\partial^2 y_{11}}{\partial t^2}\right)=0\ ;\ EA\left(\frac{\partial^2 u_{11}}{\partial {x_{11}}^2}\right)-(\rho A)\left(\frac{\partial^2 u_{11}}{\partial t^2}\right)=0 \quad (25)$$

$$EI\left(\frac{\partial^4 y_{22}}{\partial {x_{22}}^4}\right)+(\rho A)\left(\frac{\partial^2 y_{22}}{\partial t^2}\right)=0\ ;\ EA\left(\frac{\partial^2 u_{22}}{\partial {x_{22}}^2}\right)-(\rho A)\left(\frac{\partial^2 u_{22}}{\partial t^2}\right)=0 \quad (26)$$

Assuming that $y_{11}(x_{11},t)=Y_{11}(x_{11})e^{i\omega t}$ and $u_{11}(x_{11},t)=U_{11}(x_{11})e^{i\omega t}$ .The solution to above equations is in the form of :

$$Y_{11}=\sum_{n=1}^{4}R_n e^{S_n x_{11}} \quad ; \quad Y_{22}=\sum_{n=1}^{4}P_n e^{S_n x_{22}} \; ; \quad U_{11}=\sum_{n=1}^{2}J_n e^{D_n x_{11}} \; ; \quad U_{22}=\sum_{n=1}^{2}H_n e^{D_n x_{22}} \tag{27}$$

Where $S_{1,2}=\pm\left(\frac{\rho A\omega^2}{EI}\right)^{1/4}$; $S_{3,4}=\pm i\left(\frac{\rho A\omega^2}{EI}\right)^{1/4}$; $D_{1,2}=\pm i\left(\frac{\rho\omega^2}{E}\right)^{1/2}$

Now with boundary conditions and continuity equations between bonded and unbonded regions of the beam, we can obtain frequency equation.

BOUNDARY CONDITIONS AND FREQUENCY EQUATION FOR SINGLE LAP JOINT

The boundary conditions for adhesively jointed free-free beam shown in Figure 4, are as follows:

Boundary conditions for free ends of the beam at $x_{11}=0$ and $x_{22}=l$,

For adherent 1, at $x_{11}=0$: $U_{11}=0$; $\left(\frac{d^2Y_{11}}{dx^2_{11}}\right)=0$; $\left(\frac{d^3Y_{11}}{dx^3_{11}}\right)=0$ (Region 1)

For adherent 2, at $x_{22}=l$: $U_{22}=0$; $\left(\frac{d^2Y_{22}}{dx^2_{22}}\right)=0$; $\left(\frac{d^3Y_{22}}{dx^3_{22}}\right)=0$ (Region 3) (28)

For the free ends of the joint region (Region 2) at x= 0 and x= C

$$\frac{d^2Y_1}{dx^2}=0\,;\quad \frac{d^3Y_1}{dx^3}=\frac{Q^*}{EI}\,;\quad \frac{dU_1}{dx}=0\,;\quad \frac{d^3Y_a}{dx^3}=0\,;\quad \frac{d^2Y_2}{dx^2}=0\,;\quad \frac{d^3Y_2}{dx^3}=\frac{Q^*}{EI}\,;\quad \frac{dU_2}{dx}=0 \tag{29}$$

The continuity conditions of the beams –

at $x_{11}=l$; x= 0 : $\quad Y_{11}=Y_1\,;\quad \frac{d^2Y_{11}}{dx^2_{11}}=\frac{d^2Y_1}{dx^2}\,;\quad \frac{d^3Y_{11}}{dx^3_{11}}=\frac{d^3Y_1}{dx^3}-\frac{Q^*}{EI}\,;$

at $x_{22}=0$; x= C : $\quad Y_{22}=Y_2\,;\quad \frac{d^2Y_{22}}{dx^2_{22}}=\frac{d^2Y_2}{dx^2}\,;\quad \frac{d^3Y_{22}}{dx^3_{22}}=\frac{d^3Y_2}{dx^3}-\frac{Q^*}{EI}$ (30)

Where Q^* is the shear force at the jointed region, assuming shear stress τ at the free end of joint; the shear force Q^* is expresses as –

$$Q^*(x)=\frac{G_a w}{2}\left[2U_1-2U_2+t_1\left(\frac{dY_1}{dx}+\frac{dY_2}{dx}\right)\right]+G_a\frac{dY_a}{dx} \tag{31}$$

Using equations (24) and (27); the above 26 homogeneous equations, (28) to (30) may be presented in matrix form as-

[T][Z]=0 (32)

In which [T] is a [26x26] coefficient matrix and [Z] is a column vector of 26 constants as, Z_n (n=1,2….14); R_n, P_n (n=1….4); J_n, H_n (n=1,2) , for non-trivial solution of equation (32) the determinant of matrix –

$$[T] = 0 \tag{33}$$

The equation (33) is the frequency equation, the roots of which yield the complex natural frequencies ($\omega_R + i\omega_I$) of the joint system. The equation will be in terms of mechanical and geometrical properties of adherent and adhesive. The solution of equation (33) was obtained by developing code in MATLAB 7.1.

EXPERIMENTAL INVESTIGATION: RESULTS AND DISCUSSIONS

A single lap adhesively bonded joint was prepared using mild steel adherents having thickness 5 mm and width 50 mm and were joined together using epoxy resin (araldite) adhesive of 0.5 mm thickness. The overlap area was (50x100) mm^2 and length of beam L was 600 mm. The beam was suspended at each end with a thin nylon cord to simulate the free-free boundary condition in transverse vibration.

Experimentation was carried out on both monolithic beam and single lap adhesively bonded beam. Figure 5 shows the test setup for measuring the dynamic response of single lap adhesive joint and monolithic beam.

Figure 5: The Test Setup

A monolithic beam i.e. a beam possessing identical geometrical and material properties to the jointed structure but with no joint interface is shown in Figure 6,

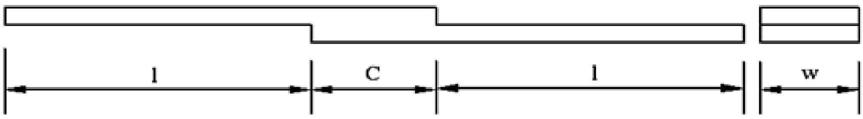

Figure 6: Monolithic Beam

Dynamic response of the bonded joint and monolithic beam was obtained by striking the beam with nylon hammer and measuring the response using accelerometer, Figure 7 shows the frequency spectrum of a bonded joint used to identify resonant frequencies; commercial package NI-LABVIEW was used to acquire response signals through Ni-DAQmx. The natural frequencies thus obtained are compared in Table 3. This preliminary experimentation was carried out to show that; adhesive joint in structure does affect the modal parameters of beam.

Detailed theoretical and experimental investigation was then carried out for single lap adhesive joint. The mechanical and geometrical properties of adhesive and adherents used in the analytical and experimental investigation are listed in Table 1 and 2 respectively.

Table 1: Mechanical Properties for Adherent and Adhesive

	E	E’	G’	ρ (Kg/m^3)	$\eta_{ae}=\eta_{ag}$
Adherent (Mild steel)	207 GPa	-----	-----	7850	----
Adhesive (Epoxy-araldite)	-----	2.46 Gpa	0.83 GPa	1150	0.1

Table 2: Geometrical Properties of the Model

h_0	l	C	w	t_1
0.5 mm	250 mm	100 mm	50 mm	5 mm

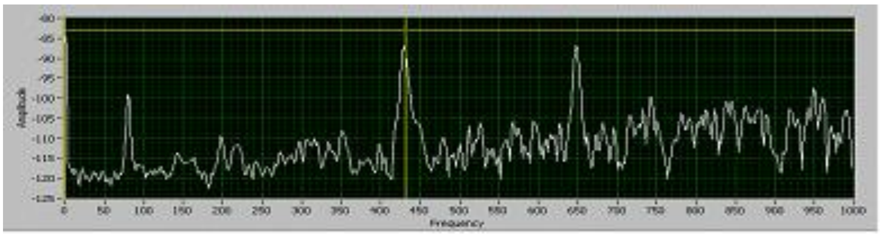

Figure 7: Frequency Spectrum of a Bonded Joint, Used to Identify Resonant Frequencies

Table 3: Natural Frequencies for Adhesive Joint and Monolithic Beam

Natural Frequencies (Hz)	Adhesively Jointed Beam (Experimental)	Monolithic Beam (Experimental)	Monolithic Beam (FEM)
ω_1	80.795	84.76	84.746
ω_2	201.325	202.649	202.85
ω_3	431.788	458.278	449.23
ω_4	660.68	665.954	666.22
ω_5	----	----	1138.5

The natural frequencies ω_R determined analytically using equation (33) for different overlap ratio (c/L). The total length L of the beam was kept constant as the lap ratio is increased. The variation of the normalized resonance frequencies $\omega^*(=\omega_R/\omega_0)$ with overlap ratio c/L is shown in Figure 8 ; where ω_0 is the fundamental natural frequency of Free-Free beam with same dimensions as beam shown in Figure 6. Three more samples of single lap adhesive joint were prepared (in addition to one for which geometrical properties are given in Table 2) with c/L=0.1,0.2 and 0.4 with same thickness of adhesive h_0=0.5 mm. The length L was kept constant equal to 600 mm for all samples.

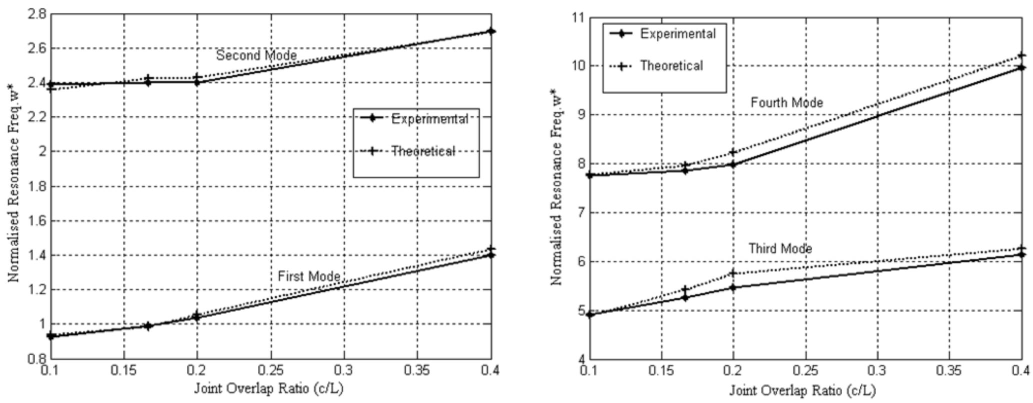

Figure 8: Variation of Normalized Resonance Frequency of Joint System with Overlap Ratio

It is evident from Figure 8 that there is increase in resonance frequency of adhesive joint system with overlap ratio. The results seem to be consistent physically; an increase in lap ratio will make the joint stiffer, thus increase the resonance frequency of the joint system. The theoretical results matches closely with experimental results, it is clear that the theoretical model derived here can be used to predict modal parameters of the adhesive joint system.

Four more samples of beam with single lap adhesive joint were prepared for experimentation having geometrical parameters as given in Table 2 and with different adhesive thickness h_0=0.5, 1.0, 2.0 and 3.0 mm. Natural frequencies are measured for each sample and also determined analytically. Figure 9 shows variation of normalized natural frequencies ω* with adhesive thickness h_0. There is decrease in normalized resonance frequency with increase in adhesive thickness indicating that the joint is becoming more flexible; the joint system stiffness reduces with increasing thickness of adhesive.

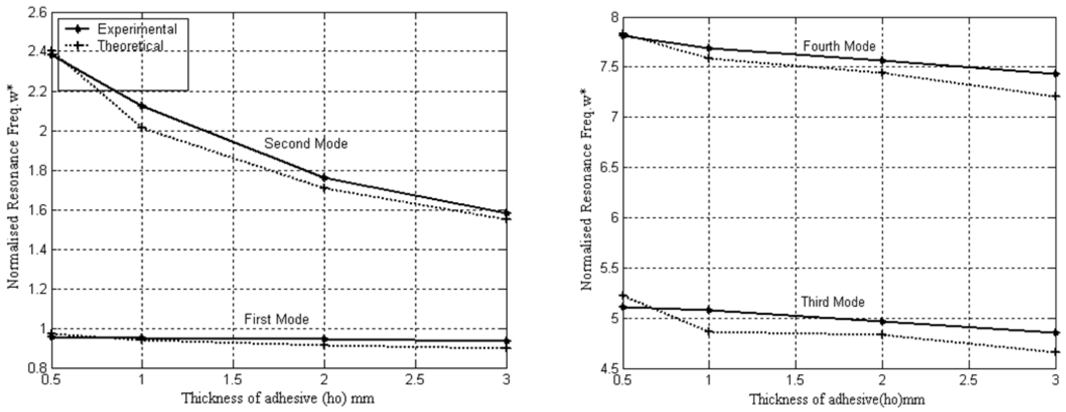

Figure 9: Effect of Adhesive Thickness on System Resonance Frequency for C/L=0.167

CONCLUSIONS

A three-parameter model of adhesive joint was investigated theoretically and experimentally to describe the behavior of the joint system. A so called third parameter y_a i.e. the deflection of mid-plane of adhesive layer was introduced in theoretical derivation of the model. Experimentation was carried out on Free-Free Euler-Bernoulli beam with single lap adhesive joint having different overlap geometry and adhesive thickness. The results of the experimentation compared with those obtained from theoretical model. Effect of joint overlap ratio and thickness of adhesive on resonance frequencies was investigated and presented in this paper.

There is increase in resonance frequency of adhesive joint system with overlap ratio. An increase in lap ratio will make the joint stiffer, thus increase the resonance frequency of the joint system. The error, particularly in higher mode is more due to systematic error in the measurement of higher natural frequencies.

The effect of adhesive thickness on the system natural frequencies presented here indicates that the system natural frequencies decrease with increasing adhesive thickness. This can be explained considering the system stiffness is reduced with increasing the adhesive thickness.

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